Universität Hamburg
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MIN-Fakultät
Fachbereich Informatik
Arbeitsbereich SAV/BV (KOGS)

Image Processing 1 (IP1)
Bildverarbeitung 1

Lecture 4 – Shape-preserving Sampling and Thresholding

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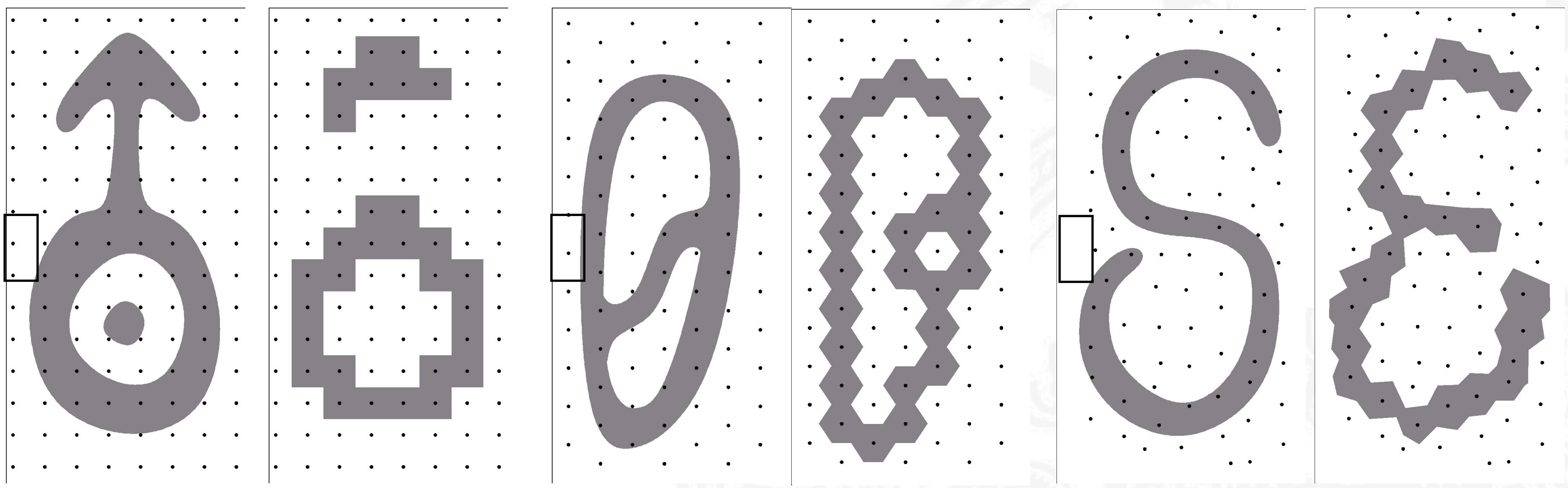
Shape-preserving Sampling of Binary Images

Problem: Shapes may change under digitization



This cannot be solved by using Shannon´s Theorem since binary images are not bandlimited.

Shape-preserving Sampling Theorem I

Shape-preserving Sampling Theorem:

The shape of an r-regular image can be correctly reconstructed after sampling with any sampling grid, if the grid point distance is not larger than r.

Stelldinger, Köthe 2003

"grid point distance": maximal distance from arbitrary sampling point to the next sampling point

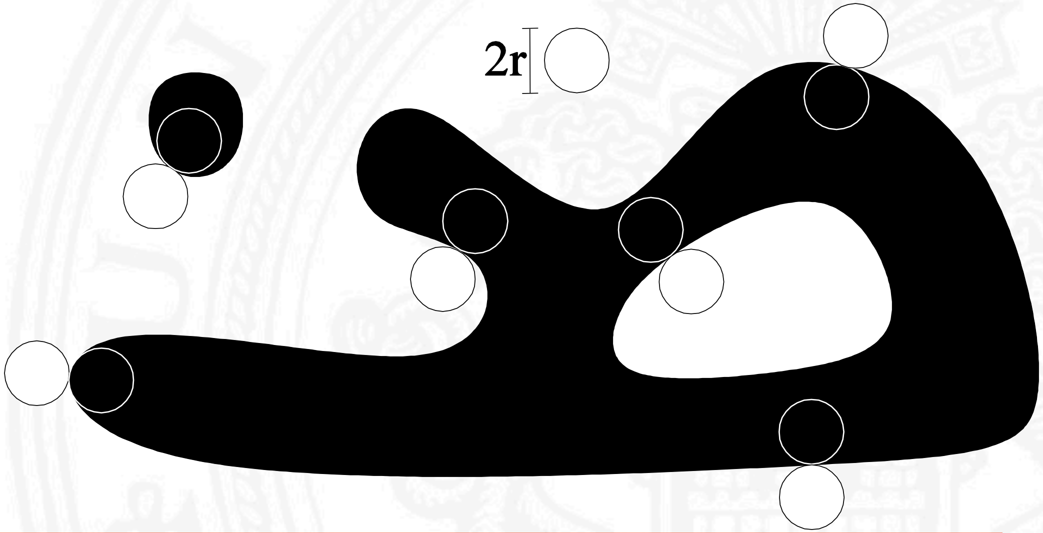
"r-regular binary image":

osculating r-discs at each boundary point of the shape

⇒ curvature bounded by 1/r

⇒ bounded thinness of parts

⇒ minimal distance between parts



Shape-preserving Sampling Theorem II

- What does correct reconstruction mean?
- Topological and geometric similarity criterion:
- One shape can be mapped onto the other by twisting the whole plane, such that the displacement of each point is smaller than r.

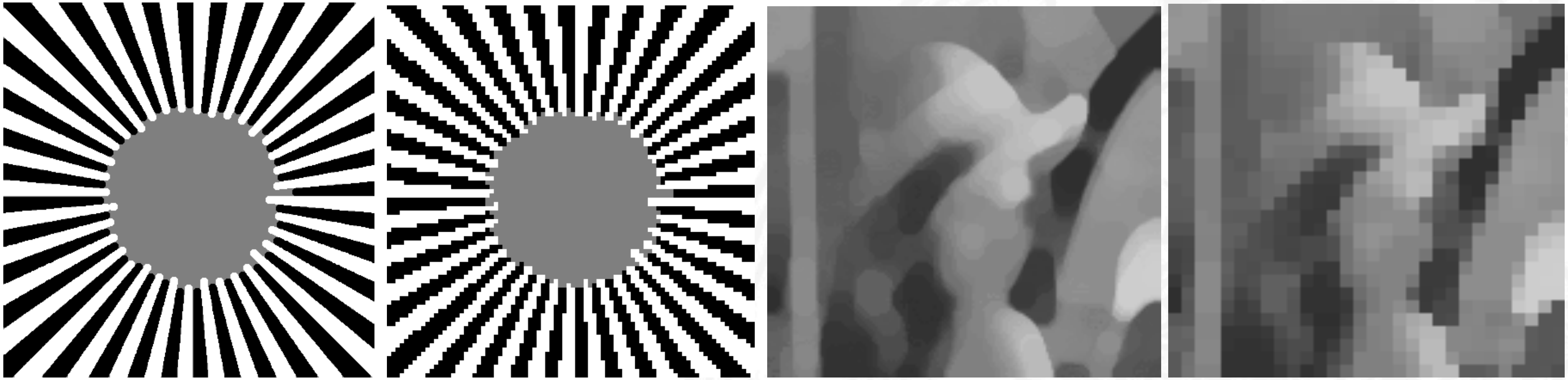
Sampling of Shapes in Arbitrary Images I

- The previous sampling theorem also holds for greyvalue images, if each greyvalue level set is an r-regular shape.
- A "level set" is the set where the image is brighter than a given threshold value.

sampling + reconstruction

Sampling of Shapes in Arbitrary Images II

- Reconstruction after sampling from r-regular originals



- The generalization to higher dimensions has been recently solved.

Comparison of the Sampling Theorems

	Shannon´s Sampling Theorem	Shape-Preserving Sampling Theorem
necessary image property	bandlimited with bandwidth W	r-regular
equation	$\left(\frac{r'}{\sqrt{2}}=\right) d < \frac{1}{2W}$	$r' < r$
reconstructed image	identical to original image	same shape as the original image
prefiltering	band-limitation: efficient algorithms (but shapes may change!)	regularization: unsolved problem
2D sampling grid	rectangular grid	arbitrary grids
dimension of definition	1D (generalizable to n-D)	2D (partly generalizable to n-D)

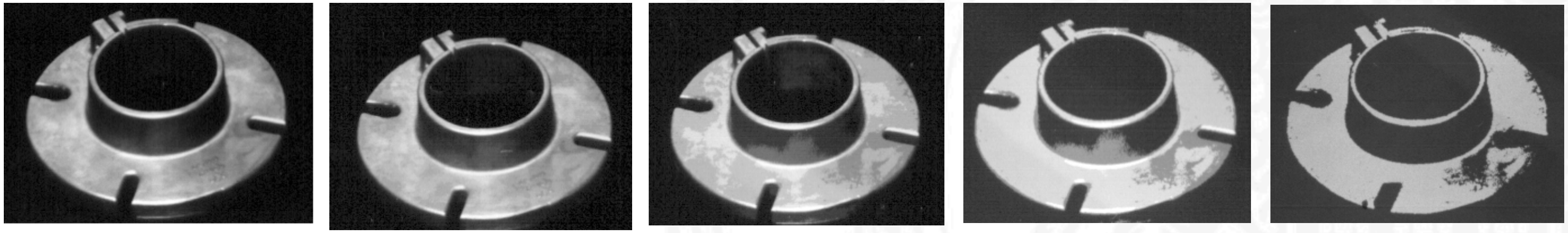
Quantization of Greyvalues

Quantization of greyvalues transforms continuous values of a sampled image function into digital quantities.

Typically 2 ... 2^{10} quantization levels are used, depending on the task.

Less than 2^9 quantization levels may cause artificial contours for human perception.

<u>Example</u>:

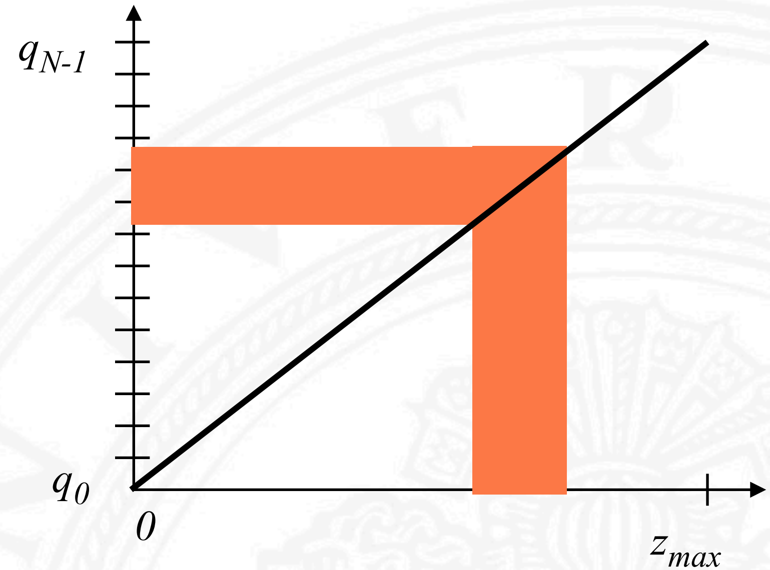


256 **16** **8** **4** **2**

Uniform Quantization

Equally spaced discrete values $q_0 \ldots q_{N-1}$ represent equal-width greyvalue intervals of the continuous signal.

Typically $N = 2^K$ for $K = 1 \ldots 10$

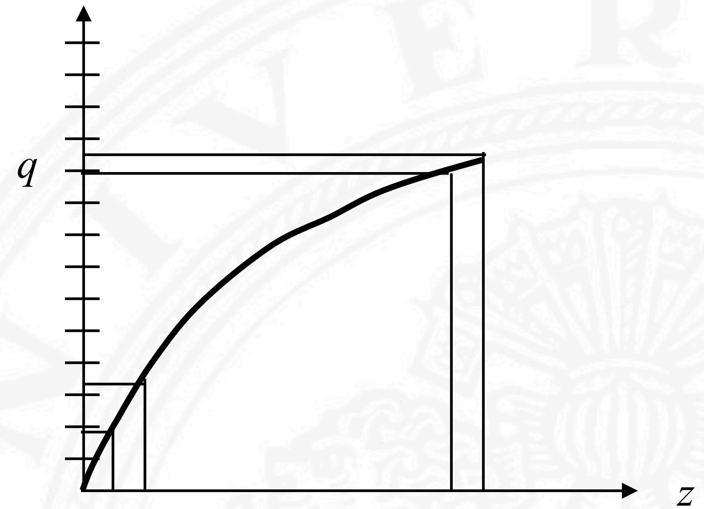


Uniform quantization may "waste" quantization levels, if greyvalues are not equally distributed.

Nonlinear Quantization Curves

E.g. fine resolution for "interesting" greyvalue ranges, coarse resolution for less interesting greyvalue ranges.

Example:
Low greyvalues are mapped into more quantization levels than high greyvalues.



Note:

Subjective brightness in human perception depends (roughly) logarithmically on the actual (measurable) brightness.

To let the computer see brightness as humans, use a logarithmic quantization curve.

Optimal Quantization I

Assumption:

Probability density p(z) for continuous greyvalues and number of quantization levels N are known.

Goal:

Minimize mean quadratic quantization error d_q by choosing optimal interval boundaries z_n and optimal discrete representatives q_n.

$$d_q^2 = \sum_{n=0}^{N-1} \int_{z_n}^{z_{n+1}} (z - q_n)^2 p(z) dz$$

Minimizing by setting the derivatives zero:

$$\frac{\delta}{\delta z_n} d_q^2 = (z_n - q_{n-1})^2 p(z_n) - (z_n - q_n)^2 p(z_n) = 0 \quad \text{for } n = 1 \text{ ... } N\text{-}1$$

$$\frac{\delta}{\delta q_n} d_q^2 = -2 \int_{z_n}^{z_{n+1}} (z - q_n) p(z) dz = 0 \quad \text{for } n = 0 \text{ ... } N\text{-}1$$

Optimal Quantization II

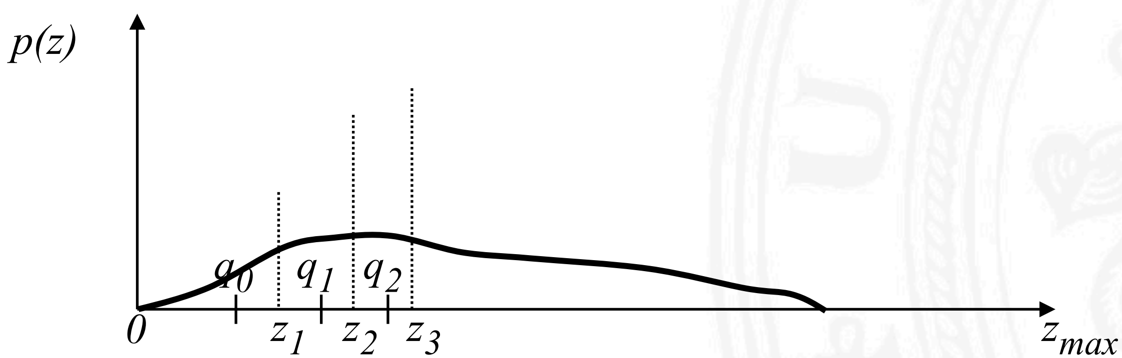
Solution for optimal quantization:

$$z_n = \frac{1}{2}(q_{n-1} + q_n) \quad \text{for } n = 1 \ldots N\text{-}1 \text{ when } p(z_n) > 0$$

Each interval boundary must be in the middle of the corresponding quantization levels.

$$q_n = \frac{\int_{z_n}^{z_{n+1}} z p(z)\, dz}{\int_{z_n}^{z_{n+1}} p(z)\, dz} \quad \text{for } n = 0 \ldots N\text{-}1$$

Each quantization level is the center-of-gravity coordinate of the corresponding probability density area.



Optimal quantization can be determined by an iterative algorithm beginning with an arbitrary choice of z_1

Binarization

For many applications it is convenient to distinguish only between 2 greyvalues, "black" and "white", or "1" and "0".

Example: Separate object from background

Binarization = transforming an image function into a binary image

Thresholding:

$$g(x,y) \Rightarrow \begin{cases} 0 & if\ g(x,y) < T \\ 1 & if\ g(x,y) \geq T \end{cases}$$

T is called „threshold"

Thresholding is often applied to digital images in order to isolate parts of the image, e.g. edge areas.

Threshold Selection by Trial and Error

A threshold, which perfectly isolates an image component must not always exist.

Selection by trial and error:

Select threshold until some image property is fulfilled, e.g.

$$q = \frac{\#\text{white pixels}}{\#\text{black pixels}} \Rightarrow q_0$$

$$\text{line strength} \Rightarrow d_0$$

$$\text{number of connected components} \Rightarrow n_0$$

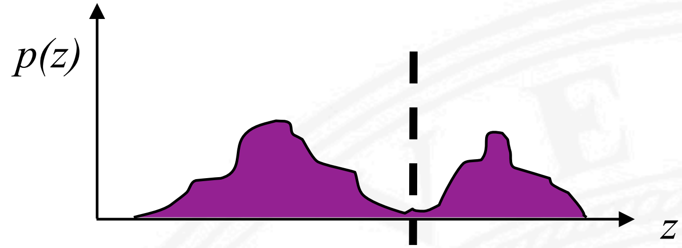
Number of trials may be small if logarithmic search can be used.

Example:

At most 8 trials are needed to select a threshold $0 \leq T \leq 255$, which best approximates a given q_0.

Distribution-based Threshold Selection

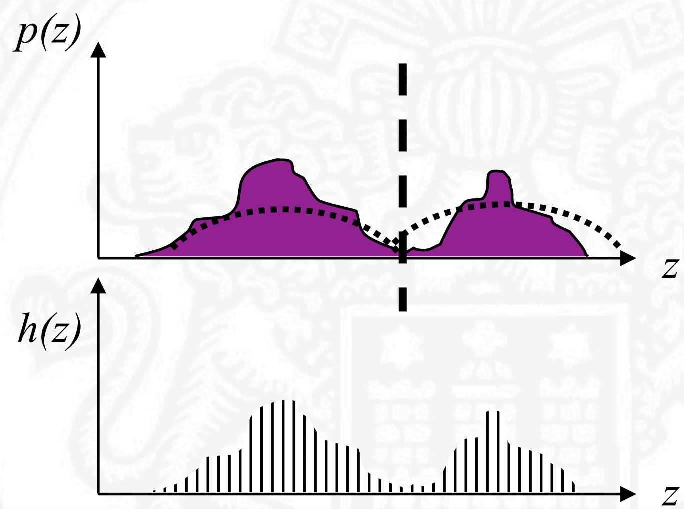
The greyvalue distribution of the image function may show a bimodality:



plausible choice of threshold

Two methods for finding a plausible threshold:

1. Find "valley" between two "hills“
2. Fit hill templates and compute intersection

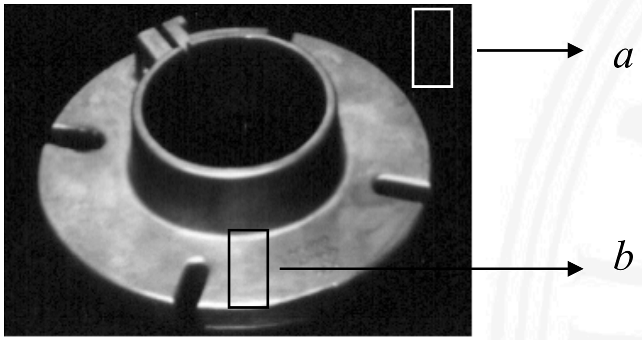


Typically, computations are based on histograms which provide a discrete approximation of a distribution.

Threshold Selection Based on Reference Positions

In many applicatons, the approximate position of image components is known a priori. These positions may provide useful reference greyvalues.

Example:



Possible choice of threshold:

$$T = \frac{a+b}{2}$$

Threshold selection and binarization may be decisively facilitated by a good choice of illumination and image capturing techniques.

Image Capturing for Thresholding

If the image capturing process can be controlled, thresholding can be facilitated by a suitable choice of

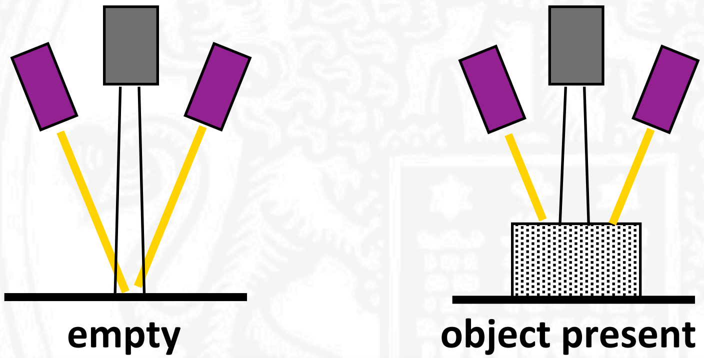
- Illumination
- camera position
- object placement
- background greyvalue or colour
- preprocessing

Example: Backlighting

Illumination from the rear gives bright background and shadowed object

Example: Slit illumination

On a conveyor belt illuminated by a light slit at an angle, elevations give rise to displacements which can be recognized by a camera.



In: Exercise 1

- Discussion and presentation of the results

Out: Exercise 2

- Brief overview of the next exercise sheet